Chapter 8

1

Conservation of energy

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Lecture 01

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Conservative Systems

2

Conservation of Energy

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- ***Energy is conserved***
  - This means that energy cannot be created nor destroyed.
  - If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer.

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Conservation of Mechanical Energy

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The mechanical energy E is the sum of the kinetic energy and potential energy:

$$E = K + U$$

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Conservation of Mechanical Energy

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"For a system with only conservative forces acting, the mechanical energy remains constant."

- $\Delta E_{mech.} = 0$
- $E_i = E_f$
- $\Delta K = -\Delta U$
- $K_i + U_i = K_f + U_f$

Water slide

Friday, 29 January, 2021 21:36

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

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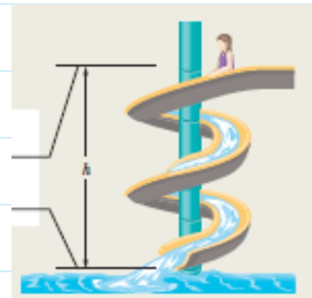
A child of mass m is released from rest at the top of a water slide, at height $h = 8.5m$ above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



Solution

Forces: Two forces act on the child. The gravitational force, a conservative force, does work on her. The normal force on her from the slide does no work because its direction at any point during the descent is always perpendicular to the direction in which the child moves.

Thus, we have only a conservative force doing work in an isolated system, so we can use the principle of conservation of mechanical energy:

$$E_{mec,b} = E_{mec,t}$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t$$

Or:

$$\frac{1}{2}mv_b^2 + mgy_b = \frac{1}{2}mv_t^2 + mgy_t$$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b)$$

Putting $v_t = 0$ and $y_t - y_b = h$leads to

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8m/s^2)(8.5m)} = 13m/s$$

This is the same speed that the child would reach if she fell 8.5 m vertically.

Ball in Free Fall

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A ball of mass m is dropped from a height h above the ground.

- Determine the speed of the ball when it is at a height y above the ground.
- Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.

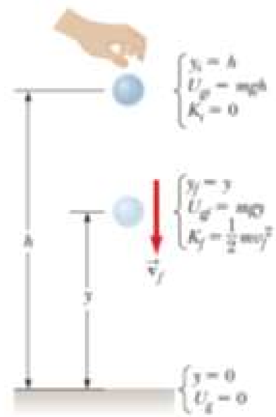

Figure 8.4 (Example 8.1) A ball is

Solution

The only force between members of the system is the gravitational force, which is conservative.

Because the system is isolated and there are no nonconservative forces acting within the system, we apply the principle of conservation of mechanical energy to the ball–Earth system. At the instant the ball is released, its kinetic energy is $K_i = 0$and the gravitational potential energy of the system is$U_{gi} = mgh$. When the ball is at a position y above the ground, its kinetic energy is $K_f = \frac{1}{2}mv_f^2$and the potential energy relative to the ground is $U_{gf} = mgy$.

Apply:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_f^2 + mgy = 0 + mgh$$

Solve for v_f:

$$v_f^2 = 2g\ h - y \rightarrow v_f = \sqrt{2g(\mathrm{h} - y)}$$

The speed is always positive. If you had been asked to find the ball's velocity, you would use the negative value of the square root as the y component to indicate the downward motion.

Solution

The initial energy includes kinetic energy equal to $\frac{1}{2}mv_i^2$.

Apply:

$$\frac{1}{2}mv_f^2 + mgy = \frac{1}{2}mv_i^2 + mgh$$

Solve forv_f:

$$v_f^2 = v_i^2 + 2g\ (h - y) \rightarrow v_f = \sqrt{v_i^2 + 2g(\ h - y)}$$

This result is valid even if the initial velocity is at an angle to the horizontal.

The Pendulum

Saturday, 30 January, 2021 15:17

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A pendulum consists of a sphere of mass m attached to a light cord of length L. The sphere is released from rest when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless.

- Find the speed of the sphere when it is at the lowest point B.
- What is the tension T_B in the cord at B?

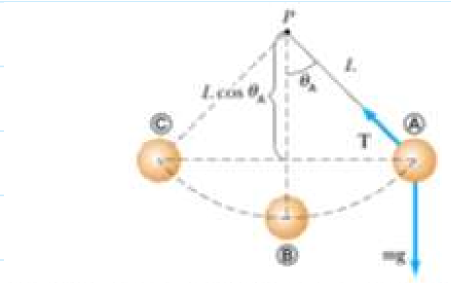

Figure 8.7 (Example 8.3) If the sphere is released

Solution

At the instant the pendulum is released, the energy of the system is entirely potential energy. At point B the pendulum has kinetic energy.

If we measure the y coordinates of the sphere from the center of rotation, then $y_A = -L\cos\theta_A$ and $y_B = -L$. Therefore, $U_A = -mgL\cos\theta_A$ and $U_B = -mgL$. Applying the principle of conservation of mechanical energy to the system gives:

$$K_A + U_A = K_B + U_B$$
$$0 - mgL\cos\theta_A = \frac{1}{2}mv_B^2 - mgL$$
$$v_B = \sqrt{2gL(1-\cos\theta_A)} \quad (1)$$

Solution

Because the force of tension does no work, we cannot determine the tension using the energy method. To find T_B, we can apply Newton's second law to the radial direction. First, recall that the centripetal acceleration of a particle moving in a circle is equal to v^2/r directed toward the center of rotation. Because $r = L$ in this example, we obtain

$$\sum F_r = T_B - mg = ma_r = m\frac{v_B^2}{L} \quad (2)$$

Substituting (1) into (2) gives the tension at point B :

$$T_B = mg + 2mg(1-\cos\theta_A)$$
$$= mg(3 - 2\cos\theta_A)$$

Exercise: A pendulum of length 2.00 m and mass 0.500 kg' is released from rest when the cord makes an angle of 30.0° with the vertical. Find the speed of the sphere and the tension in the cord when the sphere is at its lowest point.

The Spring-Loaded Popgun

Saturday, 30 January, 2021 15:18

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

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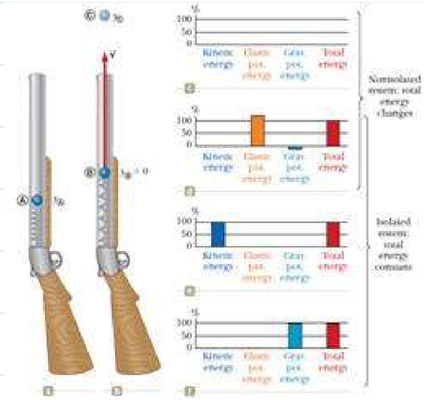
The launching mechanism of a popgun consists of a trigger-released spring. The spring is compressed to a position y_A, and the trigger is fired. The projectile of mass m rises to a position y_C above the position at which it leaves the spring, $y_B = 0$. Consider a firing of the gun for which $m = 35\ g$, $y_A = -0.12\ m$ and $y_C = 20\ m$.



- Determine the spring constant.
- Find the speed of the projectile as it moves through the equilibrium position B of the spring.

SOLUTION

Because the projectile starts from rest, its initial kinetic energy is zero. We choose the zero configuration for the gravitational potential energy of the system to be when the projectile leaves the spring. For this configuration, the elastic potential energy is also zero.

After the gun is fired, the projectile rises to a maximum height y_c. The final kinetic energy of the projectile is zero.

From the isolated system model, write a conservation of mechanical energy equation for the system between points A and C:
$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$

Substitute for each energy: $0 + mgy_C + 0 = 0 + mgy_A + \frac{1}{2}kx^2$

Solve for k: $k = \frac{2mg\, y_C - y_A}{x^2}$

Substitute numerical values:

$$k = \frac{2\ 0.035kg\ 9.80m/s^2[20.0m - -0.12m]}{0.12m^2} = 958N/m$$

SOLUTION

Analyze The energy of the system as the projectile moves through the equilibrium position of the spring includes only the kinetic energy of the projectile $\frac{1}{2}mv_B^2$. Both types of potential energy are equal to zero for this configuration of the system.

Write a conservation of mechanical energy equation for the system between points A and B :

$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$

Substitute for each energy:

$$\frac{1}{2}mv_B^2 + 0 + 0 = 0 + mgy_A + \frac{1}{2}kx^2$$

Solve for v_B:

$$v_B = \sqrt{\frac{kx^2}{m} + 2gy_A}$$

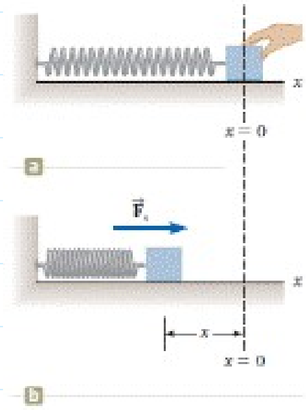
Substitute numerical values:

$$v_B = \sqrt{\frac{958N/m\ 0.120\text{m}^2}{0.035\ 0\text{kg}} + 2\ 9.80\text{m}/s^2 - 0.120\text{m}} = 19.8m/s$$

A Block–Spring System - Cons.

Saturday, 30 January, 2021 15:18

A block of mass $1.6\ kg$ is attached to a horizontal spring that has a force constant of $\kappa = 1000\ N/m$. The spring is compressed $2\ cm$ and is then released from rest. Calculate the speed of the block as it passes through the equilibrium position $(x = 0)$ if the surface is frictionless.



SOLUTION

In this situation, the block starts with $v_i = 0$at $x_i = -2.0cm$, and we want to find $v_f$at$x_f = 0$.

Use Equation 7.11 to find the work done by the spring on the system withxi_{max}:

$$\sum W_{other\ \text{forses}} = W_s = \frac{1}{2}kx_{max}^2$$

Work is done on the block, and its speed changes. The conservation of energy equation, Equation 8.2, reduces to the work–kinetic energy theorem. Use that theorem to find the speed at$x = 0$:

$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}W_s}$$

Substitute numerical values:

$$v_f = \sqrt{0 + \frac{2}{1.6kg}\left[\frac{1}{2}10000N/m0.020m^2\right]} = 0.50m/s$$

Block–Spring Collision - Cons.

Saturday, 30 January, 2021 15:19

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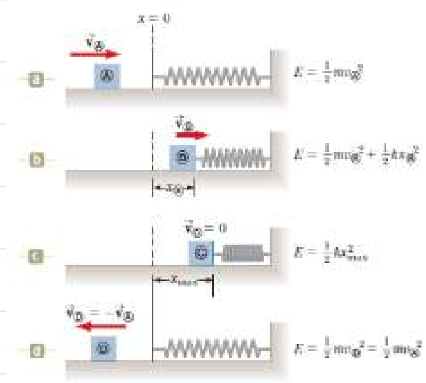
A block having a mass of $0.8\,kg$ is given an initial velocity $v_A = 1.2\ m/s$ to the right and collides with a spring whose mass is negligible and whose force constant is $\kappa = 50\ N/m$. Calculate the maximum compression of the spring after the collision.



SOLUTION

Before the collision, when the block is at A, it has kinetic energy and the spring is uncompressed, so the elastic potential energy stored in the system is zero.

After the collision, when the block is at C, the spring is fully compressed; now the block is at rest and so has zero kinetic energy.

Write a conservation of mechanical energy equation:

$$K_C + U_{sC} = K_A + U_{sA}$$

$$0 + \frac{1}{2}kx_{max}^{2\frac{1}{2}^{2}{}_{A}}$$

Solve for x_{max} and evaluate:

$$x\sqrt{\frac{m}{k}}_A\sqrt{\frac{0.80kg}{50N/m}}_{max}$$